

Value at Risk and Diversification

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Summary: A more diversified portfolio could increase the failure probability of a bank

1. Introduction

Diversification is a fundamental principle in finance. A risk averse investor with a concave utility function would always prefer a more diversified portfolio. This idea of diversification has then been transferred to the area of financial risk management and banking regulation (see the concept of „coherent“ risk measures of Artzner et al. (1998), for a critique see Rau-Bredow (2019)). It is therefore considered problematic that Value at Risk, for many years the industrial standard in financial risk management, does not always account for diversification. Here, we give a short discussion of this view.

2. Value at Risk does not always account for diversification

A classical example would be a bond trader on which a Value at Risk limit of x dollars is imposed with a confidence level p and a risk horizon T . This means that, if the portfolio remains unchanged for a period T , the probability of a loss greater than x should not be greater than $1 - p$. But the bond trader could circumvent this risk limit by buying bonds only from one issuer with a sufficiently low default probability. Value at Risk of the bond portfolio would then be zero. But if the bond trader instead diversifies and buys bonds from more and more different issuers, the probability that one of the many bond issuers defaults will at some point be greater than $1 - p$. Value at Risk will then be no longer zero, although the portfolio is now actually more diversified.

3. Value at Risk and regulatory bank capital

According to the Basel Committee of Banking Supervision, regulatory bank capital is determined in such a way that the likelihood that the bank will remain solvent over a one-year horizon is equal to 99.9%, i.e. an institution is expected to suffer losses that exceed its capital on average once in a thousand years (BCBS 2005). In other words, regulatory bank capital is equal to Value at Risk with a confidence level of 99.9% and a risk horizon of one year.

But if applied to market risk under the Basel II framework, calculation of Value at Risk is based on daily outcomes (scaled to a risk horizon of 10 days in a second step) with a much lower confidence level of 99%. This means a loss threshold is calculated which is on average exceeded every 100 trading days (approx. 5 months), whereas regulatory capital should be large enough that it is exceeded only every thousand years. It is obvious that tail risk is then only insufficiently recognized by Value at Risk. As a consequence, in order to derive the Basel regulatory capital, the 10-day Value at Risk calculated by the banks has to be multiplied by a factor of at least 3.

4. More diversification could increase the failure probability of a bank

What is the relationship between regulatory capital and the fact that Value at Risk does not always account for diversification? Diversification is often characterized as not putting all eggs into one basket. Through diversification, a high impact low probability distribution is replaced by a low impact - you will probably lose only a very few of your many baskets - high probability distribution, since if a bank diversifies across many different investments, the probability that one of these many projects goes wrong increases with the number of different projects. But for a bank with very little capital, even a relatively small loss might be sufficient to trigger a default. In such a case, more diversification could lead to a higher default probability of the bank.

Consider as an example two stochastically independent investment projects that both provide a gain of 5% with probability of 99.9% and a loss of 25% with 0.1% probability. A bank with e.g. 8% capital that invests in only one of these projects would then have a default probability of 0.1%. But if the bank instead diversifies and invests in equal parts in both projects, the default probability would almost double to $1 - 0.999^2 = 0.2\%$.

It might be argued that bank creditors (or a deposit insurance scheme) are not only concerned about the default probability of a bank, but also about the amount of losses if a default actually occurs. According to Merton (1974), creditors are short a put option on the bank's assets. Then, according to option pricing theory, a more diversified bank portfolio would lower volatility and lead to a higher market value of bank debt, whereby also reflecting higher recovery rates. But a regulator will often be primarily concerned about the default probability of a bank independently of the eventual recovery rate for bank creditors. This will in particular be the case for systemically important institutions.

5. Conclusion

The common view is that violations of the diversification principle should not be allowed for a risk measure. Contrary to that, there are cases where a more diversified portfolio increases the failure probability of a bank. This is what is reflected by the fact that more diversification could lead to a higher Value at Risk. A risk measure that, unlike Value at Risk, always accounts for diversification would then not measure risk correctly. Note that against this background, the introduction of Expected Shortfall as the new measure for market risk in the Basel framework is problematic.

In future research, this microprudential result should also be related to the macroprudential fact that if every bank holds the same broadly diversified portfolio, the system could be more vulnerable to a shock, since every bank (and not only some specialized institutions) would then be affected in the same way, potentially leading to a systemic crisis (for a discussion of this see e.g. Battiston et al. (2012)).

References

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